VCE Mathematical Methods Units 3&4 Complete Study Guide

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This guide follows the VCAA VCE Mathematics Study Design (Mathematical Methods Units 3 and 4), accredited from 2023 (version 1.1 is the current published version). At the time of writing (2026), VCAA has not published a replacement Mathematics study design, so check the VCAA Mathematical Methods page each year for any changes. It covers all four areas of study: functions, relations and graphs; algebra, number and structure; calculus; and data analysis, probability and statistics, plus exam technique for Examination 1 (technology free) and Examination 2 (CAS). Every numerical answer in this guide has been checked with computer algebra software.

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About this guide

How to use it

Each main section matches one area of study in the study design. Work through a section in this order: read the explanation, cover the worked example and try it yourself, then compare your working line by line. Finish with the "Check your understanding" questions at the end of the section. Full worked solutions for every question are in the Worked solutions section near the end.

Methods rewards two things: knowing the standard procedures so well that you never hesitate, and writing your working so that a marker can follow it. The worked examples show the level of working that earns full marks in Examination 1. For Examination 2, you still need to show the set-up (the equation you solved or the integral you evaluated) even when your CAS does the arithmetic.

Exam tip: Do the practice exam under real conditions near the end of your revision, not at the start. Use the section questions first to find your weak spots.

How Units 3&4 are assessed

Your study score is built from school-assessed coursework (SACs) and two end-of-year examinations set by VCAA.

Component	What it is	Weighting
Unit 3 school-assessed coursework	An application task: an extended investigation, usually over several lessons, focused on functions, algebra and calculus	20%
Unit 4 school-assessed coursework	Two tasks: a modelling or problem-solving task, and a second task with a strong probability and statistics focus	20%
Examination 1	Technology free, 15 minutes reading plus 1 hour writing, short and some extended answer questions; a formula sheet is provided	20%
Examination 2	CAS and one bound reference allowed, 15 minutes reading plus 2 hours writing; Section A multiple choice and Section B extended answer	40%

Both examinations cover all four areas of study. Recent Examination 1 papers have been worth 40 marks, and recent Examination 2 papers have had 20 multiple-choice questions in Section A and extended answer questions worth about 60 marks in Section B. Always check the current VCAA examination specifications and your school's SAC schedule, because details can change.

What the outcomes ask you to do

The study design lists three outcomes for Units 3 and 4. Outcome 1 is about defining and explaining key concepts and applying routine procedures. Outcome 2 is about applying

mathematical processes in non-routine contexts, including modelling and problem solving. Outcome 3 is about selecting and using technology (your CAS) to explore and solve problems. Notice that the SACs lean heavily on Outcomes 2 and 3, while the exams test all three.

A ten-week revision plan

Week	Focus	Tasks
1	Functions and graphs	Sections 1.1 to 1.4, sketch ten graphs by hand, Check your understanding 1
2	Transformations and modelling	Sections 1.5 to 1.7, build your bound reference pages for graphs
3	Algebra of functions	Composite, inverse and functional equations, Check your understanding 2
4	Equations	Exponential, log and trig equations, simultaneous equations, Newton's method
5	Differentiation	All rules until automatic, tangents and normals, Check your understanding 3
6	Applications of differentiation	Stationary points, concavity, optimisation, one past-style extended question per day
7	Integration	Antiderivatives, definite integrals, areas, average value, trapezium rule, Check your understanding 4
8	Probability	Discrete and binomial, continuous random variables, Check your understanding 5
9	Normal distribution and statistics	Sample proportions, confidence intervals, CAS drills

10	Exam practice	Practice exam under timed conditions, mark it, redo every lost mark

Contents

1. Functions, relations and graphs
2. Algebra, number and structure
3. Calculus: differentiation and its applications
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5. Data analysis, probability and statistics
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7. Practice exam
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1. Functions, relations and graphs

This area of study is about recognising the shape of each function family, knowing its key features, and describing how one graph is turned into another. Almost every extended question in both exams starts with a function, so the time you spend here pays off everywhere.

1.1 Functions, domain and range

A **relation** is any set of ordered pairs. A **function** is a relation in which each x-value is paired with exactly one y-value; graphically, every vertical line meets the graph at most once (the **vertical line test**). The **domain** is the set of x-values used and the **range** is the set of y-values produced.

VCAA writes functions in full as f: D → R, f(x) = rule, where D is the domain. If no domain is given, assume the **maximal (implied) domain**: the largest set of real numbers for which the rule is defined.

Rule contains	Restriction	Example maximal domain
√(expression)	expression ≥ 0	√(4 − x) needs $x \leq 4$
$\log_e$(expression)	expression > 0	$\log_e(2x - 1)$ needs $x > 1/2$
1 ÷ (expression)	expression ≠ 0	$1/(x + 3)$ needs $x \neq -3$
tan(expression)	expression ≠ π/2 + kπ	tan(x) needs $x \neq \pi/2 + k\pi$, $k \in Z$

Interval notation: [a, b] includes both endpoints, (a, b) excludes them, and [a, ∞) includes a and has no upper limit. Use R for all real numbers and R \ {3} for all real numbers except 3.

A function is **one-to-one** if each y-value comes from only one x-value (it passes the horizontal line test). Only one-to-one functions have inverse functions.

Hybrid functions (piecewise functions) use different rules on different parts of the domain. When sketching, use an open circle for an excluded endpoint and a closed circle for an included one.

1.2 Polynomial functions

A polynomial of degree n has the form $a_n x^n + \ldots + a_1 x + a_0$. Key features to show on any sketch: axis intercepts, stationary points and end behaviour.

Factor in rule	Behaviour at x = a
$(x - a)$	Graph cuts straight through the x-axis
$(x - a)^2$	Graph touches the x-axis (turning point)
$(x - a)^3$	Graph flattens and passes through (stationary point of inflection)

The **leading coefficient** controls end behaviour. For a positive cubic, $y \to -\infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$. For a positive quartic, $y \to \infty$ at both ends.

Useful algebra: the **factor theorem** says that if $P(a) = 0$ then $(x - a)$ is a factor of $P(x)$. For a cubic, test small integer factors of the constant term, then divide (long division or by inspection) to find the remaining quadratic factor.

Worked example 1.1. Sketch $f(x) = (x + 1)^2(x - 3)$, showing all key features.

1. x-intercepts: $(x + 1)^2(x - 3) = 0$ gives $x = -1$ (a touch point, since the factor is squared) and $x = 3$ (cuts).
2. y-intercept: $f(0) = (1)^2(-3) = -3$, so $(0, -3)$.
3. Stationary points: by the product rule, $f'(x) = 2(x + 1)(x - 3) + (x + 1)^2 = (x + 1)(2x - 6 + x + 1) = (x + 1)(3x - 5)$.
4. $f'(x) = 0$ when $x = -1$ or $x = 5/3$. $f(-1) = 0$ and $f(5/3) = (8/3)^2(-4/3) = -256/27$.
5. The cubic is positive (leading coefficient 1), so $(-1, 0)$ is a local maximum and $(5/3, -256/27)$ is a local minimum.

> **Exam tip:** On a sketch, label every key point with exact coordinates unless the question asks for decimals. A sketch with correct shape but unlabelled points usually loses marks.

1.3 Power functions

Power functions have the form $y = x^n$. The study design includes $n \in Q$ (rational powers), and you must know the shape for each type.

Type of n	Example	Shape and key features
Positive even integer	$y = x^2, x^4$	U-shaped, symmetric about y-axis, range $[0, \infty)$
Positive odd integer	$y = x^3, x^5$	S-shaped, stationary point of inflection at origin, range R
Negative odd integer	$y = x^{-1}, x^{-3}$	Hyperbola in quadrants 1 and 3, asymptotes $x = 0$ and $y = 0$

Negative even integer	$y = x^{-2}, x^{-4}$	Truncus in quadrants 1 and 2, asymptotes $x = 0$ and $y = 0$, range $(0, \infty)$
1/2, 1/4 (even root)	$y = x^{1/2} = \sqrt{x}$	Starts at origin, domain $[0, \infty)$
1/3 (odd root)	$y = x^{1/3}$	Defined for all x, vertical tangent at origin
2/3	$y = x^{2/3}$	Defined for all x, cusp at origin, range $[0, \infty)$

For $y = a(x - h)^{-2} + k$, the asymptotes are $x = h$ and $y = k$. For $y = a/(x - h) + k$ they are also $x = h$ and $y = k$.

> **Common mistake:** Writing the horizontal asymptote of $y = 2(x + 1)^{-2} - 8$ as $y = 8$. The vertical shift is -8, so the asymptote is $y = -8$.

1.4 Exponential, logarithmic and circular functions

Exponential functions. $y = Ae^{kx} + c$ has a horizontal asymptote $y = c$, y-intercept $A + c$ and range (c, ∞) when $A > 0$. If $k > 0$ the graph grows; if $k < 0$ it decays. Exponential functions are one-to-one.

Logarithmic functions. $y = A \log_e(k(x - h)) + c$ is the inverse type of an exponential. It has a vertical asymptote $x = h$. For $k > 0$ the domain is (h, ∞). There is no horizontal asymptote; the graph keeps increasing slowly.

Worked example 1.2. For $f(x) = 2 \log_e(x - 1) + 3$, find the domain, asymptote and x-intercept.

1. Need $x - 1 > 0$, so the domain is $(1, \infty)$ and the vertical asymptote is $x = 1$.
2. x-intercept: $2 \log_e(x - 1) + 3 = 0$, so $\log_e(x - 1) = -3/2$ and $x - 1 = e^{-3/2}$.
3. The x-intercept is $(1 + e^{-3/2}, 0)$. There is no y-intercept because $x = 0$ is not in the domain.

Circular (trigonometric) functions. For $y = a \sin(n(x - h)) + k$ or $y = a \cos(n(x - h)) + k$:

Feature	Value
Amplitude	abs(a)
Period	$2\pi \div n$ (for $n > 0$)
Mean (centre line)	$y = k$

Range	[k − abs(a), k + abs(a)]
Horizontal shift	h units right

For $y = a\tan(n(x - h)) + k$ the period is $\pi \div n$, there is no amplitude, and vertical asymptotes occur where $n(x - h) = \pi/2 + m\pi$, $m \in Z$.

Exact values you must know without thinking: $\sin(\pi/6) = 1/2$, $\sin(\pi/4) = \sqrt{2}/2$, $\sin(\pi/3) = \sqrt{3}/2$, $\cos(\pi/6) = \sqrt{3}/2$, $\cos(\pi/4) = \sqrt{2}/2$, $\cos(\pi/3) = 1/2$, $\tan(\pi/6) = 1/\sqrt{3}$, $\tan(\pi/4) = 1$, $\tan(\pi/3) = \sqrt{3}$. Use the unit circle and the symmetry properties $\sin(\pi - x) = \sin(x)$, $\cos(\pi - x) = -\cos(x)$, $\sin(-x) = -\sin(x)$, $\cos(-x) = \cos(x)$ to place answers in other quadrants. You also need the identities $\sin^2(x) + \cos^2(x) = 1$ and $\tan(x) = \sin(x)/\cos(x)$, and the complementary relations $\sin(\pi/2 - x) = \cos(x)$ and $\cos(\pi/2 - x) = \sin(x)$.

Worked example 1.3. Find the x-intercepts of $y = 2\sin(2x) + 1$ for $x \in [0, \pi]$.

1. Set $y = 0$: $\sin(2x) = -1/2$.
2. If $x \in [0, \pi]$ then $2x \in [0, 2\pi]$. Sine is negative in quadrants 3 and 4, and the related angle is $\pi/6$.
3. $2x = \pi + \pi/6 = 7\pi/6$ or $2x = 2\pi - \pi/6 = 11\pi/6$.
4. $x = 7\pi/12$ or $x = 11\pi/12$.

> **Exam tip:** Always change the interval for the argument before solving (here, $2x \in [0, 2\pi]$). This one habit prevents most missing-solution errors.

1.5 Transformations

The study design describes transformations from $y = f(x)$ to $y = A\,f(n(x + b)) + c$. Read the transformed rule from the inside out.

Change to the rule	Transformation	Mapping
$y = A\,f(x)$	Dilation by factor A from the x-axis (and reflection in x-axis if $A < 0$)	$(x, y) \to (x, Ay)$
$y = f(nx)$	Dilation by factor 1/n from the y-axis (and reflection in y-axis if $n < 0$)	$(x, y) \to (x/n, y)$
$y = f(x + b)$	Translation b units left (if $b >$ 0)	$(x, y) \to (x - b, y)$
$y = f(x) + c$	Translation c units up (if $c >$	$(x, y) \to (x, y + c)$

	0)	
$y = -f(x)$	Reflection in the x-axis	$(x, y) \to (x, -y)$
$y = f(-x)$	Reflection in the y-axis	$(x, y) \to (-x, y)$

The safest method for complicated cases is the **mapping method**. Write the image point as (x', y'), express the new rule in terms of x' and y', and match it to the original.

Worked example 1.4. Describe a sequence of transformations that maps $y = e^x$ onto $y = 3e^{2x - 4} + 1$.

1. Rewrite: $y = 3e^{2(x - 2)} + 1$.
2. We want x' and y' such that $y' = 3e^{2(x' - 2)} + 1$ whenever $y = e^x$. Match: $y' = 3y + 1$ and $x = 2(x' - 2)$, so $x' = x/2 + 2$.
3. The mapping is $(x, y) \to (x/2 + 2, 3y + 1)$.
4. In words: a dilation by factor 1/2 from the y-axis, a dilation by factor 3 from the x-axis, then a translation 2 units right and 1 unit up.

Common mistake: Reading $y = e^{2x - 4}$ as "translate 4 units right". You must factor first: $2x - 4 = 2(x - 2)$, so after the dilation the translation is 2 units right. Order matters: dilate then translate gives the 2-unit shift, translate then dilate would need a 4-unit shift.

Worked example 1.5. The graph of $y = x^2$ is reflected in the x-axis, dilated by factor 2 from the x-axis, then translated 3 units right and 1 unit up. Find the new rule.

Mapping: $(x, y) \to (x + 3, -2y + 1)$. So $x = x' - 3$ and $y = (1 - y')/2$. Substitute into $y = x^2$: $(1 - y')/2 = (x' - 3)^2$, giving $y' = 1 - 2(x' - 3)^2$. The new rule is $y = -2(x - 3)^2 + 1$.

1.6 Sums, differences, products and composites of functions

The study design expects you to sketch graphs of combinations such as $y = x + 1/x$, $y = e^x - x$ or $y = x \sin(x)$.

Addition of ordinates. For $y = f(x) + g(x)$, sketch both graphs lightly, then add their y-values at key points: where either graph is zero (the other graph's value is the sum), where they are equal (double the value) and where they have equal and opposite values (the sum is zero). Near an asymptote of one function, the sum follows that asymptote.

Products. For $y = f(x)g(x)$, the product is zero where either factor is zero, and the sign follows the signs of the factors. For example, $y = x \sin(x)$ is "trapped" between $y = x$ and $y = -x$ and touches those lines where $\sin(x) = \pm 1$.

Composite graphs such as $y = \sin(x^2)$ or $y = e^{\sin(x)}$ are usually sketched with CAS in

Examination 2. The key skill is describing domain, range and symmetry.

Families of functions. A family is a set of functions with one parameter, such as $f(x) = x^3 - kx$. Questions ask you to find parameter values that produce a feature: two stationary points, a given number of solutions, or a tangent through a given point. Treat the parameter as a constant, do the usual working, then solve for the parameter.

Worked example 1.6. For what values of k does $y = x^2 - 2kx + 4$ have two x-intercepts?

The discriminant $\Delta = b^2 - 4ac = 4k^2 - 16$ must be positive. $4k^2 > 16$ gives $k^2 > 4$, so $k < -2$ or $k > 2$.

1.7 Inverse graphs and modelling

The graph of an inverse function is the reflection of the original graph in the line $y = x$. Intercepts swap (an x-intercept becomes a y-intercept), asymptotes swap (a horizontal asymptote $y = c$ becomes a vertical asymptote $x = c$), and domain and range swap.

Worked example 1.7. For $f(x) = e^x - 2$, find $f^{-1}(x)$ and find where the graphs of f and f^{-1} intersect, correct to two decimal places.

1. Swap x and y: $x = e^y - 2$, so $y = \log_e(x + 2)$. $f^{-1}(x) = \log_e(x + 2)$, domain $(-2, \infty)$.
2. f is strictly increasing, so its graph and the graph of its inverse intersect on the line $y = x$. Solve $e^x - 2 = x$ with CAS.
3. $x \approx -1.84$ or $x \approx 1.15$. The points are approximately $(-1.84, -1.84)$ and $(1.15, 1.15)$.

> **Common mistake:** Solving $f(x) = x$ to find intersections of f and f^{-1} when f is decreasing. For a decreasing function, intersection points can lie off the line $y = x$. Solve $f(x) = f^{-1}(x)$ directly in that case.

Modelling. Exam contexts include tides, temperature, population growth, drug concentration and profit. Identify the parameters from the context (amplitude is half the difference between the maximum and minimum, mean value is the average of the maximum and minimum, period from the time for one cycle), and always state the answer in context with units.

Worked example 1.8. The temperature in a greenhouse is $T(t) = 18 - 6\cos(\pi t/12)$ °C, where t is hours after midnight, $0 \le t \le 24$. For how long is the temperature at least 21 °C?

1. The maximum is $18 + 6 = 24$ °C at $t = 12$ (when $\cos(\pi) = -1$).
2. Solve $18 - 6\cos(\pi t/12) = 21$: $\cos(\pi t/12) = -1/2$.
3. With $\pi t/12 \in [0, 2\pi]$, $\pi t/12 = 2\pi/3$ or $4\pi/3$, so $t = 8$ or $t = 16$.
4. The temperature is at least 21 °C for $8 \le t \le 16$, which is 8 hours.

Check your understanding 1

1. Describe a sequence of transformations that maps the graph of $y = \sqrt{x}$ onto the graph of

End of free sample

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