VCE Mathematical Methods Units 1&2 Complete Study Guide

Complete study guide · Units 1&2 · AceGuides

This guide follows the VCAA **VCE Mathematics Study Design (accredited from 1 January 2023)**, which covers Mathematical Methods Units 1–4. At the time of writing (2026), the VCAA has not published a replacement Mathematics study design, so this guide matches the study design currently in force; check the VCAA Mathematical Methods page each year for amendments. It covers every area of study in Units 1 and 2: functions, relations and graphs; algebra, number and structure; calculus; and data analysis, probability and statistics, together with the technology (CAS) skills and the application and modelling work the study design expects.

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About this guide

How to use it

Work through the sections in order the first time, because later topics (calculus, transformations of exponential and circular graphs) depend on earlier ones (functions, algebra). Each section has the same shape:

- plain-English explanations of each key idea, with definitions in **bold**
- worked examples with every step shown
- summary tables you can copy onto a revision sheet
- **Exam tip** boxes (blue) and **Common mistake** boxes (amber)
- a "Check your understanding" set at the end, with full worked solutions near the back of the guide

Cover the worked solution, attempt the question on paper, then mark yourself honestly. If you get one wrong, redo it two days later without looking.

What the study design expects

Units 1 and 2 have three outcomes, assessed across all four areas of study:

Outcome	What you must be able to do
Outcome 1	Define and explain key concepts, and apply a range of related mathematical routines and procedures
Outcome 2	Apply mathematical processes in non-routine contexts, including problem-solving, modelling or investigative tasks, and analyse and discuss these applications
Outcome 3	Select and use numerical, graphical, symbolic and statistical functionality of technology to develop mathematical ideas, produce results and carry out analysis

How Units 1 and 2 are assessed

In Units 1 and 2 your school decides how you are assessed, and results are not reported to the VCAA as scores (only S or N for each unit). Typical tasks are topic tests (often split into a technology-free part and a CAS part), an **application task** or **modelling/problem-solving task**, and end-of-semester exams. Expect your school to copy the Units 3&4 exam style, so practise that style now.

In Units 3 and 4 the structure is set by the VCAA:

Component	Format	Weighting
Unit 3 school-assessed coursework	Application task	20%
Unit 4 school-assessed coursework	Modelling or problem-solving tasks	20%
Examination 1	Technology-free, 1 hour (plus reading time)	20%
Examination 2	Technology-active (CAS), 2 hours: multiple choice plus extended response	40%

Almost everything in this guide carries straight into Units 3&4, so time spent now pays off twice.

> **Study design check:** The VCAA reviews study designs regularly. Before your exams, check the Mathematical Methods page on the VCAA website for any amendments or a newly accredited study design. The core content in this guide (functions, algebra, calculus and probability) has appeared in every Methods study design for many years and will remain relevant.

An 8-week revision plan

Week	Focus	Tasks
1	Functions and relations, algebra	Sections 1–2; all practice questions
2	Polynomial and power graphs	Section 3; sketch 10 graphs by hand
3	Transformations, exponentials and logs	Sections 4–5
4	Circular functions	Section 6; learn exact values table by heart
5	Rates of change, differentiation	Section 7
6	Antidifferentiation, probability, counting	Sections 8–9
7	Random variables, CAS skills, exam technique	Sections 10–11 and Exam technique
8	Full practice exam under timed conditions	Mark it, then redo every question you lost marks on

Contents

1. Functions and relations
2. Algebra toolkit: equations, polynomials and simultaneous equations
3. Graphs of polynomial and power functions
4. Transformations of graphs
5. Exponential and logarithmic functions

6. Circular (trigonometric) functions
7. Rates of change and differentiation
8. Antidifferentiation
9. Probability and counting
10. Introduction to discrete random variables
11. Using CAS and the application task
12. Exam technique
13. Practice exam
14. Worked solutions
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1. Functions and relations

Sets and interval notation

A **set** is a collection of objects (elements). The number sets you need are N (natural numbers 1, 2, 3, …), Z (integers), Q (rationals) and R (real numbers). R \ {2} means "all real numbers except 2". A ∪ B (union) is everything in A or B; A ∩ B (intersection) is everything in both.

Interval notation describes a continuous set of real numbers. A square bracket includes the endpoint; a round bracket excludes it.

Interval	Meaning	Number line
[a, b]	$a \le x \le b$	closed dots at both ends
(a, b)	$a < x < b$	open dots at both ends
[a, b)	$a \le x < b$	closed at a, open at b
(a, ∞)	$x > a$	open at a, arrow to the right
(−∞, b]	$x \le b$	arrow to the left, closed at b

Infinity always takes a round bracket, because it is not a number you can reach.

Relations and functions

A **relation** is any set of ordered pairs (x, y). A **function** is a relation where each x-value is paired with **exactly one** y-value. On a graph, use the **vertical line test**: if any vertical line crosses the graph more than once, it is not a function.

- $y = x^2$ is a function (each x gives one y).
- $x^2 + y^2 = 9$ (a circle) is not a function: x = 0 gives y = 3 and y = −3.
- $x = y^2$ is not a function: x = 4 gives y = ±2.

The **domain** is the set of x-values used; the **range** is the set of y-values produced. VCAA uses the notation f: D → R, f(x) = rule, where D is the domain and R is the **co-domain** (the set the outputs belong to, not necessarily the range).

A function is **one-to-one** if every y-value comes from only one x-value (it passes the **horizontal line test** as well). $y = x^3$ is one-to-one; $y = x^2$ is **many-to-one**.

Implied (maximal) domain

If no domain is stated, the **implied domain** is the largest set of x-values for which the rule is

defined. Two things restrict it in Units 1&2:

- you cannot divide by zero
- you cannot take the square root (or any even root) of a negative number

Later, logarithms add a third restriction: you can only take the log of a positive number.

Worked example 1.1. State the implied domain of $f(x) = \sqrt{(6 - 2x)} + 1/(x + 1)$.

1. Square root: $6 - 2x \geq 0$, so $x \leq 3$.
2. Denominator: $x + 1 \neq 0$, so $x \neq -1$.
3. Combine: implied domain = $(-\infty, -1) \cup (-1, 3]$.

Finding the range

Always sketch the graph over the given domain first. The range is NOT always found by substituting the endpoints, because the graph can turn around in between.

Worked example 1.2. Find the range of $f: [-1, 4) \to R$, $f(x) = x^2 - 2x - 3$.

1. Complete the square: $f(x) = (x - 1)^2 - 4$, so the turning point is $(1, -4)$. Since 1 is in the domain, the minimum is -4.
2. Endpoints: $f(-1) = 0$ and $f(4) = 5$, but $x = 4$ is excluded, so $y = 5$ is not reached.
3. The largest value approached is 5. Range = $[-4, 5)$.

> **Common mistake:** Writing the range as [0, 5) by substituting only the endpoints. Always check for a turning point inside the domain.

Hybrid (piecewise) functions

A **hybrid function** uses different rules on different parts of its domain, for example

$f(x) = x + 2$ for $x < 0$; $f(x) = 2$ for $0 \leq x \leq 3$; $f(x) = 5 - x$ for $x > 3$.

To evaluate, first decide which piece the x-value belongs to: $f(-1) = 1$, $f(3) = 2$, $f(4) = 1$. To sketch, draw each piece only over its own interval and mark endpoints with closed dots (included) or open dots (excluded). The function is **continuous** at a join if the pieces meet with no gap. Here at $x = 0$ the pieces give $0 + 2 = 2$ and 2, so they meet; at $x = 3$ they give 2 and $5 - 3 = 2$, so the graph is continuous everywhere.

Inverse functions

The **inverse** of a relation swaps x and y. Its graph is the reflection of the original in the line $y = x$. The inverse of a function f is a function (written f^{-1}) only if f is one-to-one. The domain of f^{-1} is the range of f, and the range of f^{-1} is the domain of f.

To find the rule: write $y = f(x)$, swap x and y, then make y the subject.

Worked example 1.3. f: $[2, \infty) \to \mathbb{R}$, $f(x) = (x - 2)^2 + 1$. Find f^{-1}.

1. f is one-to-one on $[2, \infty)$ (it is the right half of a parabola). Range of f = $[1, \infty)$.
2. Swap: $x = (y - 2)^2 + 1$, so $(y - 2)^2 = x - 1$ and $y - 2 = \pm\sqrt{(x - 1)}$.
3. Choose the sign using the range of f^{-1}, which is $[2, \infty)$: $y = 2 + \sqrt{(x - 1)}$.
4. f^{-1}: $[1, \infty) \to \mathbb{R}$, $f^{-1}(x) = 2 + \sqrt{(x - 1)}$.

> **Exam tip:** When you take a square root, write ± and then justify which sign you keep using the range of the inverse. Examiners award a mark for the correct domain of f^{-1}, so always state it.

Circles

A circle with centre (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$. If the equation is expanded, complete the square in x and in y.

Worked example 1.4. Find the centre and radius of $x^2 + y^2 - 6x + 4y - 3 = 0$.

1. Group: $(x^2 - 6x) + (y^2 + 4y) = 3$.
2. Complete the squares: $(x - 3)^2 - 9 + (y + 2)^2 - 4 = 3$.
3. $(x - 3)^2 + (y + 2)^2 = 16$. Centre $(3, -2)$, radius 4.

The semicircle $y = \sqrt{(r^2 - x^2)}$ is the top half of a circle centred at the origin and is a function.

Idea	Key fact
Function	each x has exactly one y (vertical line test)
One-to-one	each y from exactly one x (horizontal line test too)
Implied domain	no division by 0, no even root of a negative
Inverse	swap x and y; reflect in $y = x$; dom f^{-1} = ran f
Circle	$(x - h)^2 + (y - k)^2 = r^2$

Check your understanding 1

1. State the implied domain of $g(x) = 1/\sqrt{(x - 4)}$.
2. Find the range of f: $(-2, 3] \to \mathbb{R}$, $f(x) = 4 - x^2$.
3. f: $(-1, \infty) \to \mathbb{R}$, $f(x) = 3 - 2/(x + 1)$. Find the rule and domain of f^{-1}.
4. Find the centre and radius of the circle $x^2 + y^2 + 8x - 2y + 1 = 0$.
5. Explain, using a specific example, why $x = y^2$ is not a function.
6. $f(x) = x^2 + a$ for $x < 1$ and $f(x) = 3x - 1$ for $x \geq 1$. Find the value of a for which f is

continuous at $x = 1$.

2. Algebra toolkit: equations, polynomials and simultaneous equations

Rearranging formulas (literal equations)

To make a variable the subject, undo operations in reverse order, doing the same thing to both sides.

Worked example 2.1. Make r the subject of $V = (1/3)\pi r^2 h$, where $r > 0$.

1. Multiply by 3: $3V = \pi r^2 h$.
2. Divide by πh: $r^2 = 3V/(\pi h)$.
3. Take the positive root: $r = \sqrt{(3V/(\pi h))}$.

Quadratic equations

A quadratic $ax^2 + bx + c = 0$ can be solved by factorising, completing the square or the quadratic formula:

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$$

Worked example 2.2. Solve $x^2 - 6x + 2 = 0$ exactly by completing the square.

1. $x^2 - 6x = -2$.
2. Add (half of $-6)^2 = 9$ to both sides: $(x - 3)^2 = 7$.
3. $x - 3 = \pm\sqrt{7}$, so $x = 3 \pm \sqrt{7}$.

The discriminant

The **discriminant** is $\Delta = b^2 - 4ac$. It tells you how many real solutions a quadratic has (and how many times the parabola meets the x-axis).

Discriminant	Number of real solutions	Graph
$\Delta > 0$	two distinct	crosses x-axis twice
$\Delta = 0$	one (repeated)	touches x-axis at turning point
$\Delta < 0$	none	does not meet x-axis
Δ a perfect square (a, b, c	two rational solutions	factorises over Q

rational)		

Worked example 2.3. For what values of m is the line $y = mx - 1$ a tangent to $y = x^2 + 3$?

1. Equate: $x^2 + 3 = mx - 1$, so $x^2 - mx + 4 = 0$.
2. A tangent touches once, so $\Delta = 0$: $m^2 - 4(1)(4) = 0$.
3. $m^2 = 16$, so $m = 4$ or $m = -4$.

Exam tip: "Intersects", "touches" and "does not meet" questions are discriminant questions in disguise: set the two expressions equal, rearrange to = 0, then use $\Delta > 0$, $\Delta = 0$ or $\Delta < 0$.

Polynomials, division, and the remainder and factor theorems

A **polynomial** is an expression $a_n x^n + \ldots + a_1 x + a_0$ where n is a whole number. Its **degree** is the highest power; the **leading coefficient** is a_n.

Long division: $P(x) \div (x - a)$ gives a quotient $Q(x)$ and a remainder R, with $P(x) = (x - a)Q(x) + R$.

Worked example 2.4. Divide $x^3 + 2x^2 - 5x + 1$ by $x - 2$.

1. $x^3 \div x = x^2$. Multiply: $x^3 - 2x^2$. Subtract: $4x^2 - 5x$.
2. $4x^2 \div x = 4x$. Multiply: $4x^2 - 8x$. Subtract: $3x + 1$.
3. $3x \div x = 3$. Multiply: $3x - 6$. Subtract: remainder 7.
4. So $x^3 + 2x^2 - 5x + 1 = (x - 2)(x^2 + 4x + 3) + 7$.

The **remainder theorem**: the remainder when $P(x)$ is divided by $(x - a)$ is $P(a)$. Check: $P(2) = 8 + 8 - 10 + 1 = 7$. Dividing by $(bx - a)$ gives remainder $P(a/b)$.

The **factor theorem**: $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.

Worked example 2.5. Factorise $P(x) = 2x^3 - 3x^2 - 11x + 6$ and solve $P(x) = 0$.

1. Try factors of the constant term: $P(3) = 54 - 27 - 33 + 6 = 0$, so $(x - 3)$ is a factor.
2. Divide (or compare coefficients): $P(x) = (x - 3)(2x^2 + 3x - 2)$.
3. Factorise the quadratic: $2x^2 + 3x - 2 = (2x - 1)(x + 2)$.
4. $P(x) = (x - 3)(2x - 1)(x + 2)$, so $x = 3$, $1/2$ or -2.

Worked example 2.6. When $P(x) = x^3 + ax^2 - 4x + 5$ is divided by $x + 2$, the remainder is 11. Find a.

1. Remainder $= P(-2) = -8 + 4a + 8 + 5 = 4a + 5$.
2. $4a + 5 = 11$, so $a = 3/2$.

Common mistake: Dividing by $x + 2$ means substituting $x = -2$, not $x = 2$. Set the divisor

End of free sample

This is 10 of 56 pages.

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